

# GPS and the geoid

by Prof. Charles Merry, University of Cape Town

GPS provides full 3D positions. However, the heights use the WGS84 ellipsoid as a reference surface, whereas the user almost invariably requires heights above sea level (the geoid).

This article discusses the relationships between GPS heighting and the geoid and the status of geoid modelling, with particular reference to southern Africa.

GPS (and other GNSS systems) provides fully three-dimensional positions – latitude, longitude and height. The reference surface used is the WGS84 ellipsoid, and heights are heights above this surface – ellipsoidal heights. Due to a combination of unmodelled residual refraction and weak geometry (satellites below the horizon are not visible) GPS-determined heights are generally less accurate than horizontal positions (by a factor ranging from 1,5 to as much as 3).

Nevertheless, there are many applications of GPS where heights are useful, if not essential, especially in engineering surveys. However, the type of height that is needed is a height above mean sea level (MSL) – commonly called orthometric height. To be more specific, the reference surface employed for heighting is the geoid, a level surface that corresponds very closely with MSL. Due to inhomogeneities in the density of the Earth's crust and upper mantle the geoid departs from the ellipsoid by up to 120 m. This separation is known as geoidal height (or geoidal undulation) –  $N$  (see Fig. 1).

In order to convert GPS-derived ellipsoidal heights to orthometric heights we need an accurate model of geoidal heights. This is where the difficulty arises – it is extremely difficult to determine such a model, or at least to determine an accurate model. In the next section we will discuss some of these methods and their limitations.

### Geoid modelling

In essence, there are only two methods – geometric and gravimetric:

#### Geometric

This approach is the one most favoured for GPS surveys of small extent (area covered less than 10 - 20 km<sup>2</sup>).

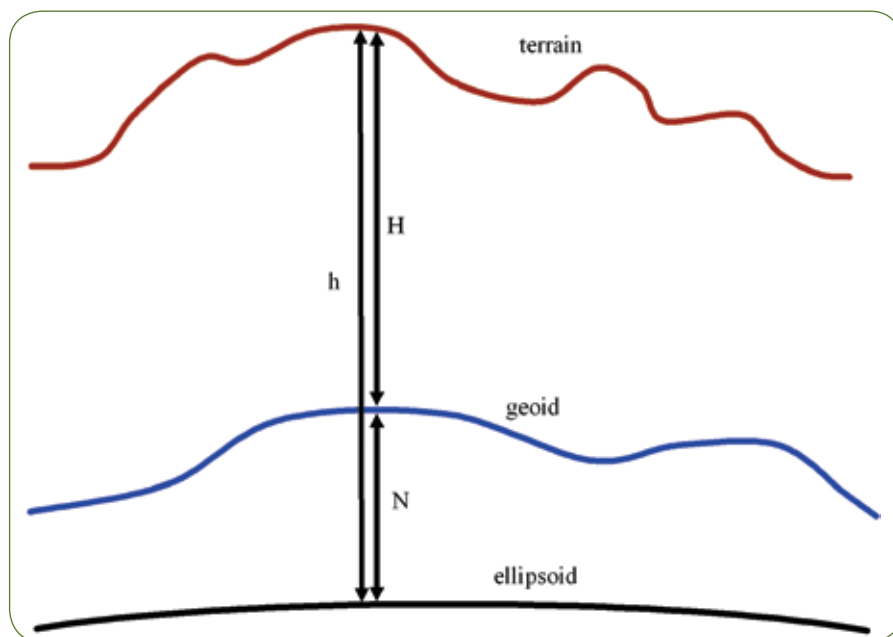


Fig. 1: Geoidal height.

GPS measurements are taken at benchmarks with known orthometric height  $H$ . The difference between the GPS-derived ellipsoidal height  $h$  and the orthometric height  $H$  provides the geoidal height  $N$  at that point (see Fig. 1):

$$N = h - H$$

In the simplest case a single control point would provide a constant height shift. However, the most common case involves three benchmarks at which GPS measurement are made. This allows two tilts (North-South and East-West) and a shift to be determined. In effect, the geoid is modelled by a tilted plane (Fig. 2). These tilts and shift can then be applied to the heights determined from GPS at any new points in the area. The "calibration" of RTK GPS prior to the start of a survey incorporates this model, as well as a model for horizontal shifts and a rotation in azimuth.

It is possible to take GPS measurements at more than three benchmarks. In

this case the commercial software will use these data to get a more reliable model of the tilted plane (i.e. the geoid is still assumed to take this shape over the region of interest). It is possible to use the extra data to generate a more complex model for the geoid, using for example polynomials, splines or Kriging, but this will require some expertise on the part of the user.

The chief limitations of the geometric case are:

- The model is only valid over the area encompassed by the known benchmarks (extrapolation beyond this area is highly inadvisable).
- The simple tilted plane model can only be safely used over a small area (the geoid is much more complex in shape).
- It is not always possible (or convenient) to find sufficient benchmarks with known orthometric heights in the area of interest.

#### Gravimetric

This approach provides a uniform grid of geoidal heights over a large area, in contrast to the scattered point values

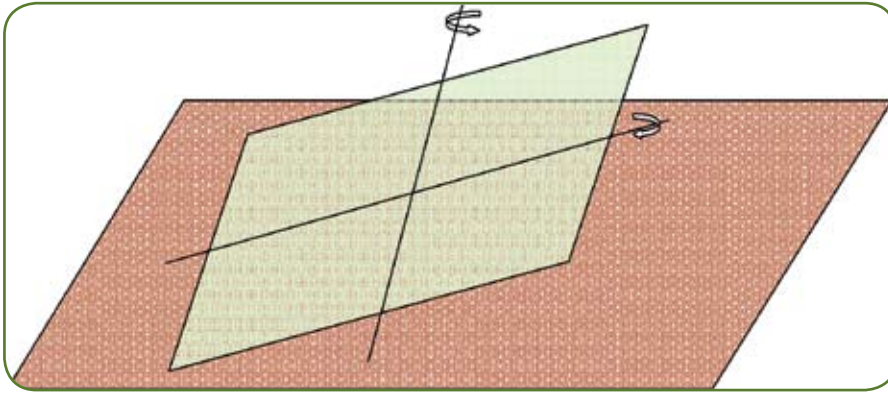


Fig. 2: Tilted plane model of the geoid.

over a limited area provided by the geometric method. However, it requires a complex numerical integration of gravity anomalies to determine a geoidal height. The key formula is that due to Stokes:

$$N = \frac{R}{4\pi\gamma} \iint \Delta g \cdot S(\psi) \cdot d\sigma$$

Without going into the details of the other terms, the core "observation" is the gravity anomaly  $\Delta g$ , which represents the difference between observed gravity and a theoretical gravity value. The integration takes place over the entire surface of the Earth, so that to get a single geoidal height tens of thousands of gravity observations are needed. The gravity anomaly also requires a correction for the effect of terrain variations, so that a detailed digital elevation model is also needed.

An alternative representation of the geoid is to use a spherical harmonic expansion of the Earth's geopotential (a spherical harmonic model is basically a two-dimensional Fourier series). Typically the coefficients of this model are determined from the analysis of the perturbations in the orbits of low-orbiting satellites. Nowadays it is common to use these two representations together in one of two ways:

- Use the satellite-based spherical harmonic expansion to determine the long wavelength (low frequency) component of the geoid, and a modified version of Stokes' formula to determine the high frequency component, integrating over a spherical "cap", not the entire Earth.
- Use both satellite data and terrestrial gravity anomalies in a single solution for a high-order spherical harmonic expansion of the Earth's gravity field. This requires significant computer resources to compute

the coefficients, and is beyond the capability of all but the largest agencies.

Although the gravimetric approach provides good spatial coverage over large areas and can provide the detailed structure of the geoid, it suffers from some drawbacks:

- It is computationally intensive and mathematically complex.
- The results are only as good as the underlying gravity measurements. If there are errors in the gravity data, there will be errors in the geoid. More seriously, there are many parts of the world where gravity data are sparse or non-existent, leading to gaps or smoothing in the geoid model.
- The gravimetric geoid is susceptible to biases and tilts, due to errors in satellite orbit modelling and to gaps in terrestrial gravity data sets.

**Combination approach**

It is possible to combine the gravimetric and geometric approaches. The gravimetric geoid is computed first, and then "calibrated" by using GPS-determined geoidal heights at discrete points over the entire region. This calibration would, in the simplest case, consist of a bias and two tilts. However, much more complex correction surfaces are possible, and in the case of continental geoid models desirable. This approach also compensates for any tilts or biases that may exist in the precise levelling networks and enables the user to transform his GPS-derived heights directly to the national levelling datum.

**Current geoid models**

Until very recently the standard model for the geoid has been the Earth Geopotential Model 1996

(EGM96), developed jointly by the US Department of Defense's National Geo-Spatial Intelligence Agency (NGA), NASA and the Ohio State University [1]. A simplified gridded version of EGM96 is embedded in most handheld GPS receivers to enable the user to get "heights above MSL" from his GPS (at an accuracy level of a few metres). EGM96 is also incorporated in most GPS processing packages used for surveying. It uses a spherical harmonic expansion to degree 360 (137 000 coefficients), based upon a combination of satellite tracking data and terrestrial gravity anomalies.

The expansion to degree 360 means that the effective "wavelength" of this geoid model is one degree - i.e. any variation in the geoid shape smaller in extent than one degree (about 100 km) will not be modelled. In addition, because of gaps and errors in the data, the inherent accuracy of this model is only around 40 cm (worse over large parts of Africa). This is not as bad as it sounds, as precise GPS surveying uses differential GPS and correspondingly it is differences in geoidal height that are important. EGM96 can represent these with an accuracy of around 10 cm plus 2 parts per million (ppm) of the distance between points.

At the University of Cape Town (UCT) we have used EGM96 as a basis for more detailed models of the geoid over Africa. Our approach is the first of the combination methods mentioned in the previous section (EGM96 provides the low frequency contribution, terrestrial gravity anomalies in Stokes' formula provide the high frequency contribution).

More recently we have replaced the EGM96 spherical harmonic expansion with that deduced from the GRACE satellite mission [2]. GRACE consists of a pair of satellites in a low orbit — small variations in the range between them are a measure of changes in the Earth's gravity field [3]. GRACE has improved our knowledge of the low frequency coefficients of this field by an order of magnitude over EGM96. The UCT model (AGP2007) provides a detailed 5' grid of the geoid over Africa. However, its accuracy and detail are limited in areas where no gravity data exist.

Earlier this year NGA introduced its new geopotential model – EGM2008 [4]. This model, based upon a combination of GRACE data and

terrestrial gravity anomalies, is a spherical harmonic representation of the geopotential to degree 2190 (over 4,7-million coefficients). A contour representation of this model for southern Africa is given in Fig. 3.

EGM2008 is more detailed and more accurate than EGM96. It is difficult to say with any certainty how accurate it is, but some initial testing indicates that the accuracy could be better than 10 cm over North America and Europe. In South Africa, some limited testing (using GPS measurement at precise levelling benchmarks) shows an accuracy of around 15 cm. It could be much worse in other parts of Africa.

Fig. 4 shows the difference between the EGM2008 and EGM96 geoid models in southern Africa. For most of the region the differences are small and reflect the improvement achieved by using a more detailed representation. However, in central Mozambique the differences reach 3 m and cover a large region. Correspondence with the developers of EGM2008 (S. Kenyon, personal communication, 2007) indicates that (for this model) terrestrial gravity anomalies were predicted in central Mozambique using a digital elevation model. No measured gravity anomalies were used. Whether this approach is correct remains to be seen – the shape of the geoid in central Mozambique as shown by EGM2008 may not be real. It would be very interesting to see what would result from taking GPS measurements at levelling benchmarks (if there are any) in that region.

Two of the goals of the Chief Directorate: Surveys and Mapping (CDSM) in modernising the national height network are to re-adjust the precise levelling network and to provide an accurate calibrated geoid model for all users. The production of a set of calibration points (GPS at levelling benchmarks) for this purpose is almost complete (R. Wonnacott, personal communication, 2008).

**Conclusions**

Surveyors will only be able to take full advantage of the third dimension of GPS if they have an accurate and reliable model of the geoid. In small regions with an adequate supply of height control points it is possible to use the geometric approach and get good results. Where height control

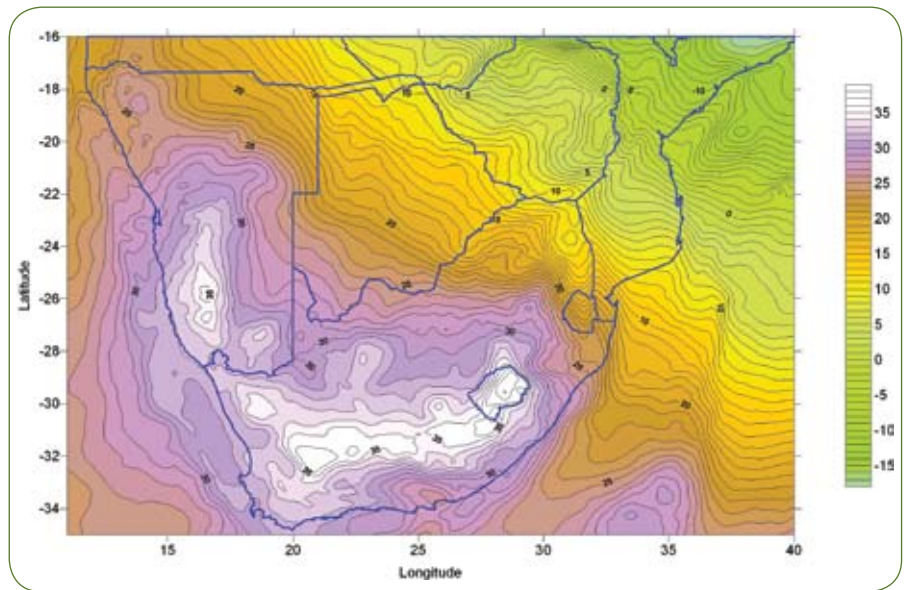


Fig. 3: EGM2008 geoid model for southern Africa.

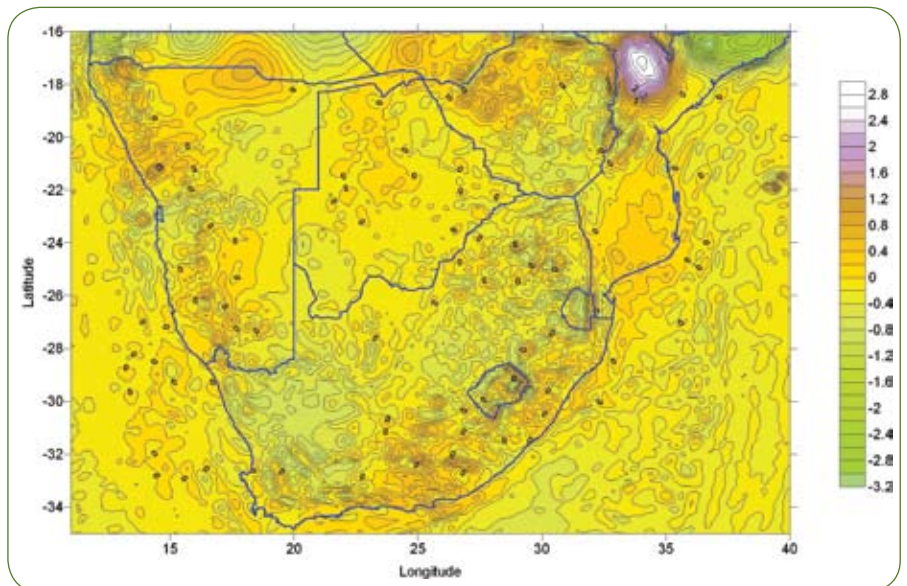


Fig. 4: EGM2008 minus EGM96 geoid models for southern Africa.

is inadequate or non-existing then a good gravimetric model is essential. Although substantial improvements in modelling the geoid using gravity data have been made, there is still room for further improvement.

**References**

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Contact Prof. Charles L Merry, School of Architecture, Planning & Geometrics, UCT, Tel 021 650-3576, charles.merry@uct.ac.za ©